

Quark-hadron duality of nucleon spin structure function

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Abstract. Bloom-Gilman quark-hadron duality of nuclear spin structure function is studied by comparing the integral of g_1 from perturbative QCD prediction in the scaling region to the moment of g_1 in the resonance region. The spin structure function in the resonance region is estimated by the parametrization forms of non-resonance background and of resonance contributions. The uncertainties of our calculations due to those parametrization forms are discussed. Moreover, the effect of the $\Delta(1232)$ -resonance in the first resonance region and the role of the resonances in the second resonance region are explicitly shown. Elastic peak contribution to the duality is also analyzed.

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1 Introduction

Recently, the new evidence of valence-like quark-hadron duality in the nucleon unpolarized structure function F_2 was reported by Jefferson Lab. [1]. It is claimed that the new data of F_2 can revisit the quark-hadron duality and the duality is valid even in the rather low momentum transfer region of $Q^2 \sim 1 \text{ GeV}^2$. In addition, ref. [1] also showed that the duality holds even for individual resonance contribution as well as for the entire inelastic resonance region.

We know Bloom-Gilman quark-hadron duality [2] tells that prominent resonances do not disappear relative to background contribution even at a large Q^2 and moreover, the average of the oscillate resonance peaks in the resonance region is the same as that of the scaling structure function at a large Q^2 value. The origin of Bloom-Gilman quark-hadron duality has been discussed by De Rujula, Georgi and Politzer [3] with a QCD explanation. It was extensively analyzed by Carlson and Mukhopadhyay [4–6] through a consideration of the asymptotic perturbative QCD (pQCD) behaviors of the resonance electromagnetic transition amplitudes at large Q^2 (see also ref. [7]). The quark-hadron duality of the nucleon structure function F_2 for different targets, like proton, deuteron and light complex nuclei, was also discussed by Ricco *et al.* and by Simula *et al.* [8–10]. In the papers of refs. [8–10] it is concluded that the elastic and $\Delta(1232)$ peaks lead to remarkable vi-

olations of the duality for the nucleon unpolarized structure function F_2 . Recently, Close and Isgur [11] investigated Bloom-Gilman duality by using the quark model. They reiterated that the quark-hadron duality is essential to understand the physics behind the connection between pQCD and non-perturbative QCD (nQCD). Moreover, they stressed that the duality shows fundamental issues in the strong interaction. Till now, many interesting studies of the quark-hadron duality have appeared [12–18].

It is known that Bloom-Gilman quark-hadron duality for the nucleon spin structure function g_1 is still an open question, because we are short of data in the resonance region. One naturally expects that the onset of the quark-hadron duality for g_1 of the proton target is at a larger Q^2 than the valid value of the onset of the duality for the proton unpolarized structure function F_2 [6]. This is because that very strong Q^2 -dependence of g_1 at low Q^2 is needed by the well-known Gerasimov-Drell-Hearn (GDH) sum rule [19]. An experimental study of the quark-hadron duality in the nucleon spin structure function g_1 was performed by HERMES group very recently [20]. The data indicate that the onset of the duality for g_1 is believed at a larger Q^2 than 1.7 GeV^2 .

In this work, we will employ the parametrization form of the nucleon spin structure function, which was given by Simula *et al.* in ref. [10], to represent g_1 in the resonance region. Then, we shall compare the integral of the nucleon spin structure function in the resonance region to the one from pQCD prediction of the scaling spin

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structure function in the scaling region in order to check Bloom-Gilman quark-hadron duality of g_1 . In addition, the uncertainties of our calculations will be shown.

2 Calculations and results

The parametrization form of the nucleon spin structure function in the resonance region, in ref. [10], contains nucleon elastic effect and the contributions of the resonances and non-resonance background. The last one is smoothly parametrized from the Regge behavior, expected to be dominant at low values of Q^2 , to the partonic description valid at high values of Q^2 . This approach for the parametrization of the background was inspired by the work of ref. [21]. Fourteen parameters are involved in this form. Those parameters have been fixed and explicitly given in ref. [10]. It is claimed that the total fit of all the 209 experimental data points can be achieved and the obtained χ^2 is about 0.66. Moreover, a simple Breit-Wigner shape was used to describe the W -dependence of the contribution of an isolated resonance following the approaches of refs. [22,23]. All four-star resonances, having a mass $M_R < 2 \text{ GeV}$ and a total transverse photoamplitude $\sqrt{|A_{1/2}|^2 + |A_{3/2}|^2}$ larger than $0.05 \text{ GeV}^{-1/2}$, are considered. The relativistic Breit-Wigner shape and the interference between the longitudinal and transverse cross-sections are also included for each resonance. It should be mentioned that the GDH sum rule value at the real photon point of the proton target of this parametrization form is $204 \pm 23 \mu\text{b}$ and the forward spin polarizability γ_0 is $(-45 \pm 20) \times 10^{-6} \text{ fm}^4$.

Conventionally, the entire resonance region with the center-of-mass energy (W) being $M + m_\pi \leq W \leq 2 \text{ GeV}$ (M and m_π are the masses of the nucleon and pion meson, respectively) is divided into several parts including the first resonance region with $M + m_\pi \leq W \leq 1.38 \text{ GeV}$, the second one with $1.38 \leq W \leq 1.61 \text{ GeV}$, and the third one with $1.61 \leq W \leq 1.80 \text{ GeV}$. We know that the first resonance region is $\Delta(1232)$ -resonance dominant, the second one mainly contains the contributions from the resonances of negative parity $D_{13}(1520)$, $S_{11}(1535)$ and positive parity $N^*(1440)$. It is believed that the contribution from the two negative-parity resonances is more important than the role of the Roper resonance $N^*(1440)$. Moreover, $S_{11}^*(1650)$, $D_{15}(1680)$, $F_{15}(1680)$ and $D_{33}(1700)$ are involved in the third resonance region. Beyond these three resonance regions, $F_{35}(1905)$ and $F_{37}(1950)$ are considered for the main contribution of the resonances with $W \leq 2 \text{ GeV}$. Therefore, the ten resonances considered in the parametrization of g_1 in ref. [10] reasonably represent almost all the contributions of the resonances.

First of all, let us calculate the average of the structure function g_1 in the entire inelastic resonance region

$$I^r(Q^2) = \int_{\xi_{\min}}^{\xi_{\max}} d\xi g_1^r(x_B, Q^2), \quad (1)$$

where r stands for the parametrization of g_1 in the resonance region (see ref. [10] in detail), and the Bjorken

variable x_B is

$$x_B = \frac{Q^2}{2M\omega}, \quad (2)$$

with $\omega = \frac{W^2 - M^2 + Q^2}{2M}$ being the photon energy. In eq. (1) ξ is the Nachtmann variable [24]

$$\xi = \frac{2x_B}{(1 + \sqrt{1 + 4M^2x_B^2/Q^2})}, \quad (3)$$

and ξ_{\min} (ξ_{\max}) in the integrated interval of eq. (1) stands for ξ with the minimum of the center-of-mass energy $W = M + m_\pi$ (the maximum of $W = 2 \text{ GeV}$). One can also estimate the average of the resonance contribution with respect to ξ in each individual resonance region of W . It should be mentioned that the Nachtmann variable is the correct one in studying QCD scaling violations in the nucleon because it partially takes the target-mass correction into account [25]. For a detailed study of the target-mass correction on the moments of g_1 the interested reader is referred to ref. [26].

We know that Bloom-Gilman quark-hadron duality indicates that the smooth scaling curve seen at high Q^2 region is an accurate average over the resonance bumps seen at low Q^2 . To check the duality, we compare the average in eq. (1) to the integral of the scaling spin structure function in the deep inelastic scattering region

$$I^S(Q_0^2) = \int_{\xi_{\min}}^{\xi_{\max}} d\xi g_1^S(x_B, Q_0^2) \quad (4)$$

at a high Q_0^2 , where S stands for the scaling structure function. ξ_{\min} and ξ_{\max} in eq. (4) are the same as those in eq. (1). Therefore, the related center-of-mass energy W in eq. (4) is much larger than that in eq. (1) because of large Q_0^2 . Here, we choose $Q_0^2 = 30 \text{ GeV}^2$, so that the scaling structure function is mainly related to pQCD physics.

Then, we can compare the scaling behavior of $I^S(Q_0^2)$ at a high Q_0^2 value (say $Q_0^2 \sim 30 \text{ GeV}^2$) to the average of g_1 (see eq. (1)) in the resonance region including the resonance bumps at a low Q^2 to check the occurrence of the quark-hadron duality for the nucleon spin structure function g_1 . We know that the constancy of the ratio

$$R(Q^2) = \frac{I^r(Q^2)}{I^S(Q_0^2)} \quad (5)$$

is a test of the quark-hadron duality. In fig. 1, we display the ratio $R(Q^2)$ of the average of g_1 in the entire inelastic resonance region including the resonance peaks to the integral of the next-to-leading order pQCD prediction of the scaling spin structure function. The scaling structure function by Glück, Reya, Stratmann and Vogelsang (GRSV) [27] is used here. One can also calculate the ratio $R(Q^2)$ with some other pQCD prediction of the scaling spin structure function like the one by the Asymmetry Analysis Collaboration (AAC) [28]. Our results show that the two ratios are very similar in this kinematical region and they both are strongly Q^2 -dependent

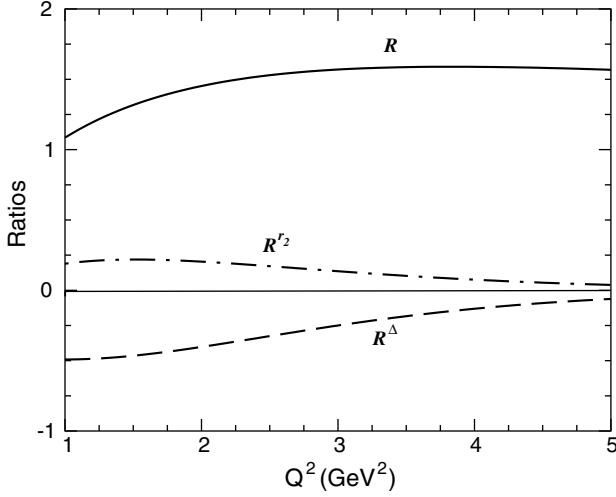


Fig. 1. Ratios with the scaling structure function of GRSV. The solid curve is the ratio in the entire inelastic resonance region. The dashed and dotted-dashed curves are the ratios $R^\Delta(Q^2)$ and $R^{r2}(Q^2)$, respectively, contributed by the $\Delta(1232)$ -resonance in the first resonance region and by the resonances in the second resonance region.

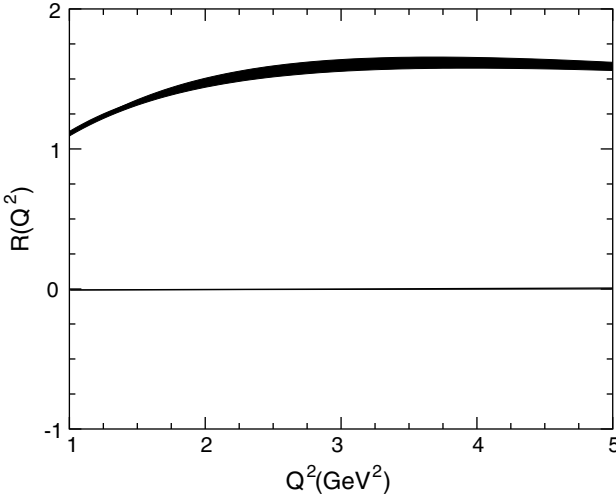


Fig. 2. The shaded band of the ratio $R(Q^2)$ with the scaling structure function of GRSV. The width is due to the uncertainties of the parametrization form of g_1 in the resonance region.

in the region of $Q^2 \leq 2 \text{ GeV}^2$ (see fig. 1). They become slightly Q^2 -dependent when $Q^2 \geq 2 \text{ GeV}^2$. It implies that the quark-hadron duality for g_1 may preserve in the range $Q^2 \geq 2 \text{ GeV}^2$. In addition, in fig. 1, we also separately plot the roles of the $\Delta(1232)$ -resonance in the first resonance region by

$$R^\Delta(Q^2) = \frac{\int_{\Delta\xi_1} d\xi g^\Delta(x_B, Q^2)}{\int_{\Delta\xi_1} d\xi g^S(x_B, Q_0^2)}, \quad (6)$$

and of all the resonances in the second resonance region by

$$R^{r2}(Q^2) = \frac{\int_{\Delta\xi_2} d\xi g^{r2}(x_B, Q^2)}{\int_{\Delta\xi_2} d\xi g^S(x_B, Q_0^2)}. \quad (7)$$

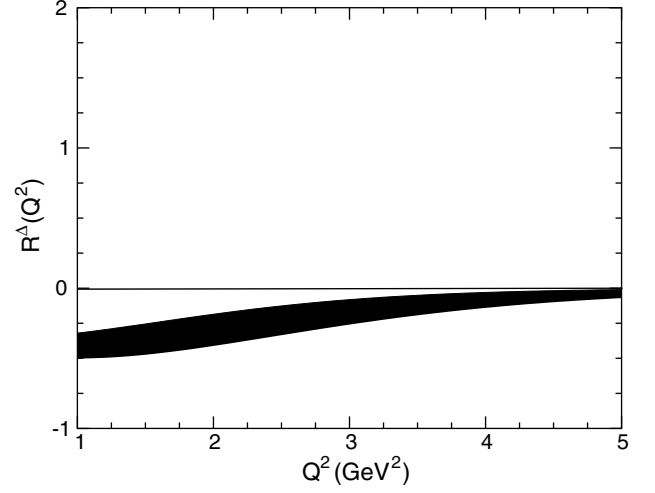


Fig. 3. The shaded band of the ratio $R^\Delta(Q^2)$ with the scaling structure function of GRSV. Notations as in fig. 2.

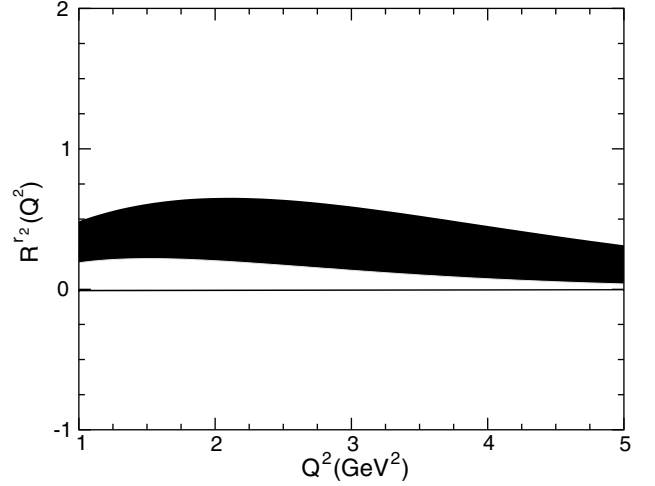


Fig. 4. The shaded band of the ratio $R^{r2}(Q^2)$ with the scaling structure function of GRSV. Notations as in fig. 2.

In eqs. (6) and (7), $\Delta\xi_i$ stands for the interval of ξ with W in the i -th resonance region, and g^Δ and g^{r2} are the spin structure functions contributed by the $\Delta(1232)$ -resonance and by all the resonances, like $S_{11}(1535)$, $D_{13}(1520)$ and $N^*(1440)$, in the second resonance region. They can also be estimated by the parametrization forms of the resonance electromagnetic transition amplitudes in ref. [10]. We can clearly see, from fig. 1, that the role of $\Delta(1232)$ violates the quark-hadron duality more evidently than that of the resonances in the second resonance region. This phenomenon is reasonable because the transition amplitudes of the $\Delta(1232)$ -resonance provide a negative contribution to the ratio. Moreover, from our numerical calculations we find that the non-resonance background contribution is also important even in the first resonance region. In addition, the two ratios by eqs. (6) and (7) with the AAC pQCD calculation are also the same as those with GRSV scaling structure function. To consider the uncertainties of our calculations explicitly, we, in fig. 2, show the ratio

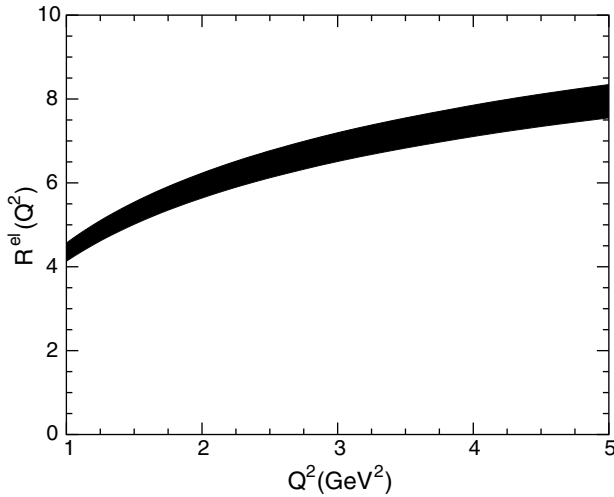


Fig. 5. Ratio $R^{\text{el}}(Q^2)$ of GRSV. The shaded band is due to the uncertainties of the parametrization forms of the nucleon elastic form factors.

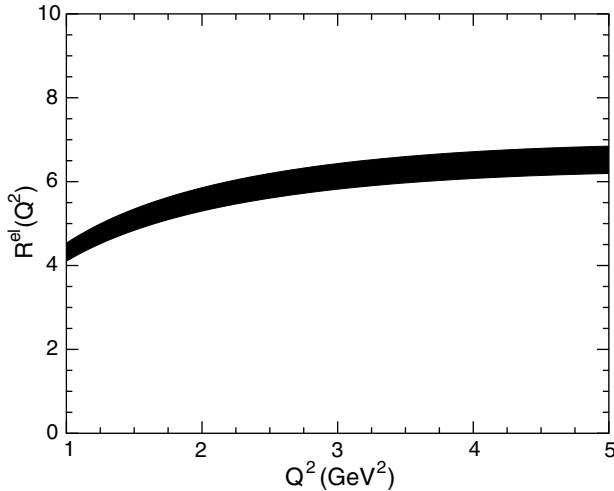


Fig. 6. Ratio $R^{r2}(Q^2)$ of AAC. Notations as in fig. 5.

$R(Q^2)$ of GRSV including the uncertainties of the pseudo-data of g_1 in the resonance region (see ref. [10] in detail). In the same way, figs. 3 and 4 show the uncertainties of our calculations of the $\Delta(1232)$ -resonance contribution and of the one of the resonances in the second resonance region.

Besides the comparison of the integrals of the nucleon spin structure functions in the inelastic resonance region and in the scaling one, the study of the local Bloom-Gilman quark-hadron duality of g_1 in the elastic region is also of a great interest. In fact, there are several works which have stressed the important contribution of the elastic peak to the nucleon spin structure functions [29], to the GDH sum rule, and to Bloom-Gilman quark-hadron duality of F_2 [12,30] in the literature. Here, in order to check the local duality of g_1 in the elastic region, we calculate a ratio

$$R^{\text{el}}(Q^2) = \frac{I^r(Q^2; \text{el})}{I^S(Q_0^2; \text{el})}, \quad (8)$$

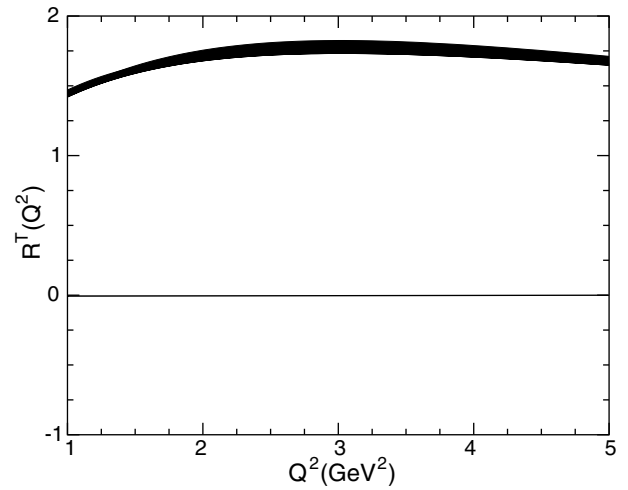


Fig. 7. Total ratio $R^T(Q^2)$ of GRSV. The shaded band indicates the uncertainties in the parametrization form of g_1 in the resonance region.

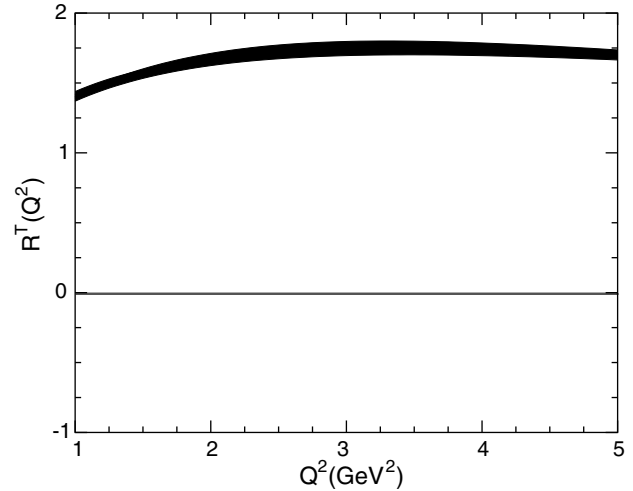


Fig. 8. Total ratio $R^T(Q^2)$ of AAC. Notations as in fig. 7.

where

$$I^r(Q^2; \text{el}) = \int_1^{\xi_\pi} d\xi g_1^r(x_B, Q^2) = \frac{G_M(Q^2)(G_E(Q^2) + \tau G_M(Q^2))}{2(1 + \tau)} \frac{\xi_{\text{el}}^2}{2 - \xi_{\text{el}}} \quad (9)$$

is the contribution of the nucleon form factor with $\tau = Q^2/4M_N^2$ and $\xi_{\text{el}} = 2/(1 + \sqrt{1 + 1/\tau})$. $I^S(Q_0^2; \text{el})$ in the denominator of eq. (8) represents the integral of the scaling structure function g_1^S with the same interval of ξ as in eq. (9). It is an integral in unphysical region as discussed in detail in ref. [30]. In the work of ref. [10], the parametrization forms of the nucleon elastic form factors by Mergell, Meissner and Drechsel [31] is employed, assuming a 5% uncertainty. Here, we also set $Q_0^2 = 30 \text{ GeV}^2$. The ratios obtained are shown in figs. 5 and 6, respectively, with GRSV and AAC scaling structure functions including the uncertainties of the nucleon form factors as well. On the one hand, we see that the elastic peak violates

duality remarkably because of the explicit Q^2 -dependence of the ratio of GRSV (see fig. 5). It implies that in the range of $Q^2 \leq 5 \text{ GeV}^2$, the local duality of g_1 in the elastic region does not survive. On the other hand, the ratio of AAC in fig. 6 becomes slightly Q^2 -dependent when $Q^2 \geq 3 \text{ GeV}^2$. The large difference between the two curves of GRSV and AAC is due to the different behaviors of the two scaling spin structure functions in the large- ξ region. It is expected that the future precise measurements of the nucleon structure function in the large- ξ region can clarify this issue.

Furthermore, figs. 7 and 8, respectively, show a total ratio $R^T(Q^2)$,

$$R^T(Q^2) = \frac{I^r(Q^2; \text{el}) + I^r(Q^2)}{I^S(Q_0^2; \text{el}) + I^S(Q_0^2)}, \quad (10)$$

including the uncertainties of g_1 in the resonance region and in the elastic region. Two scaling structure functions of GRSV and AAC are considered separately. It should be mentioned that $R^T(Q^2)$ includes the roles of the elastic peak in the elastic region and of g_1 in the inelastic resonance region simultaneously. One may find that the quark-hadron duality still holds when $Q^2 \geq 2 \text{ GeV}^2$. This conclusion is similar to the one for $R(Q^2)$ in the total inelastic resonance region as shown in figs. 1 and 2.

3 Conclusions

By using the phenomenological parameterization form of the nucleon spin structure functions g_1 , we have studied the onset of Bloom-Gilman quark-hadron duality. We employed the pQCD prediction and the parametrization form to represent the nucleon spin structure function g_1 in the scaling region and in the resonance region, respectively. We estimate the uncertainties of our calculations. These uncertainties are due to the parametrization form of g_1 in the resonance region of ref. [10]. The ratios between the average of g_1 in the entire inelastic resonance region to the integrals of it in the scaling region are displayed. We also explicitly show the roles of the $\Delta(1232)$ -resonance and of the resonances in the second resonance region. The elastic peak contribution is illustrated in our calculations as well. The ratios of the entire inelastic resonance region show that the duality may preserve when $Q^2 \geq 2 \text{ GeV}^2$. Moreover, we display that the effect of the $\Delta(1232)$ -resonance violates quark-hadron duality. The role of the elastic peak to the local duality is inconclusive because of the large difference of the two ratios in figs. 5 and 6. Our calculations also indicate the important effect of the non-resonance background. The ratios in figs. 7 and 8 include the total contributions of the elastic peak and of g_1 in the inelastic resonance region. They tell that the onset of the quark-hadron duality may still appear when $Q^2 \geq 2 \text{ GeV}^2$. In addition, the similar behaviors of the total ratios of GRSV and AAC in figs. 7 and 8 indicate that the duality may hold when $Q^2 \geq 2 \text{ GeV}^2$. The two figures also show that the large difference between the elastic contributions $R^{\text{el}}(Q^2)$ of GRSV and AAC is reduced.

In fact, there was a discussion about the contribution of the elastic peak to the local duality of F_2 (see refs. [32–34]). It is found that the elastic local duality becomes more and more sensitive to the behavior of the scaling structure function at large ξ when Q^2 increases. So far, whether the elastic contribution satisfies the local duality of F_2 is still an open question. We need high-precision scaling structure functions, especially in the large- ξ region, to clarify this issue. It should be mentioned that the almost Q^2 -independences of the ratios at $Q^2 \geq 2 \text{ GeV}^2$ shown in figs. 1, 2 and 4 evidently deviate from unity. It is argued that this feature may be due to the difficulty in accurately modeling the large- ξ behavior of g_1 , since there is a very limited amount of data in the deep inelastic scattering region available particularly in the large- ξ or large- x_B region. So far, how to assume the extrapolation from the measured region to the elastic one in the perturbative QCD next-to-leading order predictions is not clearly. Moreover, the extraction of the structure function depends on the ratio of the longitudinal-to-transverse cross-sections. In fact, this ratio is quite uncertain and not small in the large- x_B and low- Q^2 regions. Thus, to test duality in this region by experiments, we need high-precision data in which the longitudinal and transverse cross-sections can be well separated.

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